

Mindfulness:

Choose Your Breath:

Initial (focus on breathing)

Heart/Belly

Calming (2 in, 4 out)

Energizing (4 in, 2 out)

Bell Ringer:

The sum of 4 consecutive integers is -2.

What are the integers?

| | | |
|-------------|----|----------------------|
| $a = n$ | -2 | $a + b + c + d = -2$ |
| $b = n + 1$ | -1 | |
| $c = n + 2$ | 0 | |
| $d = n + 3$ | 1 | |
| $4n + 6$ | | |

$$4n + 6 = -2 - 6$$

$$4n = -8$$

$$n = -2$$

1. Set up let statements for appropriate expressions and using these expressions set up an equation that allows you to find each number described. Be sure to find EACH integer you are looking for.

- (a) Find two consecutive integers such that ten more than twice the smaller is seven less than three times the larger.

14 $2n + 10 = 3(n+1) - 7$

15 $2n + 10 = 3n - 4 + 4$

- (b) Find two consecutive even integers such that their sum is equal to the difference of three times the larger and two times the smaller.

$a = n$
 $b = n + 2$

$2n + 14 = 3n - 2n$
 $14 = n$

$a + b = 3b - 2a$

$2n + 2 = 3(n+2) - 2n$

$2n + 2 = 3n + 6 - 2n$

$2n + 2 = n + 6$

$-n - 4 = -4$

$n = 4$

4
6

(c) Find three consecutive integers such that three times the largest increased by two is equal to five times the smallest increased by three times the middle integer.

$$\begin{aligned}
 a &= n & 3c+2 &= 5a+3b \\
 b &= n+1 & 3(n+2)+2 &= 5n+3(n+1) \\
 c &= n+2 & 3n+6+2 &= 5n+3n+3 \\
 & & \cancel{3n}+8 &= 8n+3
 \end{aligned}$$

$$\begin{aligned}
 \frac{5n}{5} &= \frac{5}{5} \\
 n &= 1, 2, 3 \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \\
 & \quad n+1 \quad n+2
 \end{aligned}$$

(d) Find three consecutive odd integers such that the sum of the smaller two is three times the largest increased by seven.

$$\begin{aligned}
 n &= a \\
 n+2 &= b \\
 n+4 &= c
 \end{aligned}$$

$$\begin{aligned}
 -17 \\
 -15 \\
 -13
 \end{aligned}$$

$$\begin{aligned}
 a+b &= 3c+7 \\
 \cancel{n+2} &= 3n+12+7 \\
 -2n-15 &= -2n-19 \\
 -17 &= n
 \end{aligned}$$

$$\begin{array}{r}
 -17 \\
 +15 \\
 \hline
 -32 \\
 \checkmark
 \end{array}$$

$$\begin{array}{r}
 -13 \\
 -3 \\
 \hline
 -39 \\
 +7 \\
 \hline
 -32 \\
 \checkmark
 \end{array}$$

APPLICATIONS

$$a = n, b = n + 2, c = n + 4$$

3. In an opera theater, sections of seating consisting of three rows are being laid out. It is planned so each row will be two more seats than the one before it and 90 people must be seated in each section. How many people will be in the third row?



$$a + b + c = 90$$

$$3n + 6 = 90 - 6$$

$$3n = 84 \quad n = 28$$

$$\begin{array}{r} 28 \\ + 2 \\ \hline 30 \\ + 2 \\ \hline 32 \end{array}$$

4. In the same opera theater balcony sections with three rows of seating are being mapped as well. In these sections there must be an odd number of seats in each row and each row must have two more seats than the one before it. The last stipulation is that the front row must have one quarter the total seats in the back 2 rows combined. How many seats will be in each row?

$$n, n + 2, n + 4$$

$$\frac{1}{4}(n + 2 + n + 4) + 2n + 6 = 3n + 6$$

$$\frac{1}{2}n + \frac{3}{2} + 2n + 6 = 3n + 6$$

$$2\frac{1}{2}n + 7.5 = 3n + 6$$

$$\cancel{2\frac{1}{2}n} + 1.5 = 3n - 2\frac{1}{2}n$$

$$1.5 = \frac{1}{2}n$$

$$n = 1.5 \times 2$$

$$n = 3$$

MUR

REASONING

$$a=n, b=n+3, c=n+6$$

5. Instead of finding even or odd consecutive integers we could also look for integers that differ by a number other than 2. Find three numbers that each differ by 3 such that 5 times the largest integer is equal to three times the smallest increased by 5 times the middle.

$$5(n+6) = 3n + 5(n+3)$$

$$5n+30 = 3n+5n+15$$

$$5n+30 = 8n+15$$

$$-3n \quad -15$$

$$n = 5$$

$$n+3 = 8$$

$$n+6 = 11$$

6. What do you think every other even integer means? Set up a let statement that would show this.

2, 4, 6, 8, 10, 12, 14, 16, 18

$$a=n, b=n+4, c=n+8$$

7. Find three every other even integers such that the sum of all three is equal to three times the largest decreased by the other two numbers.

$$3n+12 = 3(n+8) - (n+n+4)$$

$$3n+12 = 3n+24 - 2n - 4$$

$$3n+12 = n+20-12$$

$$3n = n+8$$

$$2n = 8$$

$$\frac{2n}{2} = \frac{8}{2}$$

$$n = 4$$

$$n+4 = 8$$

$$n+8 = 12$$

Exercise #3: When $2(x-h)+k=8$ is solved for x in terms of h and k , its solution is which of the following?
 Show the algebraic manipulations you used to get your answer.

(1) ~~$4+k-k$~~

(2) $h+4-\frac{k}{2}$

(3) ~~$k-\frac{h}{2}+8$~~

(4) ~~$4-h+k$~~

~~$2(x-h) = 8-k$~~

~~$x-h = \frac{8-k}{2}$~~
 ~~$x = \frac{8-k}{2} + h$~~

Many times this technique is used when we want to **rearrange** a **formula** to solve for a **quantity of interest**.

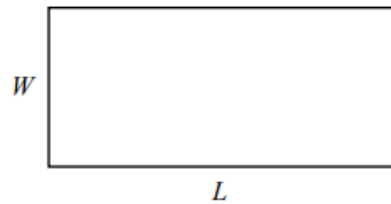
Exercise #4: For a rectangle, the **perimeter**, P , can be found if the two dimensions of length, L , and width, W , are known.

$$P = 2L + 2W$$

- (a) If a rectangle has a length of 12 inches and a width of 5 inches, what is the value of its perimeter? Include units.

$$P = 2(12) + 2(5) = 24 + 10 = 34$$

$(P = 34 \text{ inches})$



- (b) Write a formula for the perimeter, P , in terms of L and W .

$$P = 2(L + W), P = 2L + 2W$$

- (c) Rearrange this formula so that it "solves" for the length, L . Determine the value of L when $P = 20$ and $W = 4$.

$$P = 2L + 2W \rightarrow P - 2W = 2L \rightarrow L = \frac{P - 2W}{2}; L = \frac{P}{2} - W$$

$$L = \frac{20}{2} - 4 = 10 - 4 = 6$$

$$P = 2L + 2W$$

$$20 = 2(6) + 2(4)$$

$$20 = 2(6 + 4)$$

$$20 = 2(10)$$

$$20 = 20 \checkmark$$

There is one last complication that we need to look at that is often challenging for students at all levels. Let's take a look at this in the next problem.

Exercise #5: Consider the equation $ax+b=cx+d$. We'd like to solve this equation for x . Let's start with the situation where we know the values of a , b , c and d .

(a) Solve: $8x+1=5x+22$

$$\begin{array}{r} 8x = 5x + 21 \\ -5x \quad -5x \\ \hline 3x = 21 \\ \frac{3x}{3} = \frac{21}{3} \quad x = 7 \end{array}$$

(b) Now solve: $ax+b=cx+d-b$

$$\begin{array}{r} ax = cx + d - b \\ -cx \quad -cx \\ \hline ax - cx = d - b \\ x(a-c) = d - b \\ \frac{x(a-c)}{a-c} = \frac{d-b}{a-c} \quad x = \frac{d-b}{a-c} \end{array}$$

Exercise #6: Which of the following solves the equation $ax-k=3(x+h)$ for x in terms of a , k , and h . Show the manipulations to find your answer.

(1) $\frac{3h+k}{a-3}$

(3) $\frac{k+3h}{a+3}$

(2) $\frac{3a+k}{h-1}$

(4) $\frac{h+3}{a+k}$

$$\begin{array}{r} ax - k = 3x + 3h + k \\ -3x \quad -3x \\ \hline ax - 3x = 3h + k \\ x(a-3) = 3h + k \\ \frac{x(a-3)}{a-3} = \frac{3h+k}{a-3} \\ x = \frac{3h+k}{a-3} \end{array}$$