FLUENCY

1. Write the following products as polynomials in either x or t. The first is done as an example for you.

(a) $5x(2x-4)$

$=(5x)(2x)-(5x)(4)$

$=(5\cdot 2)(x\cdot x)-(5\cdot 4)(x)$

$=10x^2-20x$

(b) $3t(t+7)$

$3t^2+21t$

(c) $-4x(5x+1)$

$-20x^2-4x$

(d) $4(t^2-5t+2)$

$4t^2-20t+8$

(e) $x(x^2-2x-3)$

x^3-2x^2-3x

(f) $-5t(2t^2+3t-7)$

$-10t^3-15t^2+35t$

2. Perhaps the most important type of polynomial multiplication is that of two binomials. Make sure you are **fluent** with this skill. Write each of the following **products** as an **equivalent polynomial** written in **standard form**. The first problem is done as an example using **repeated distribution**.

(a) $(x+5)(x-3)$

$=(x+5)(x)+(x+5)(-3)$

$=(x)(x)+(5)(x)+(x)(-3)+(-5)(3)$

$=x^2+5x-3x-15$

$=x^2+2x-15$

(b) $(x-10)(x-4)$

$x^2-14x+40$

(c) $(x+3)(x+12)$

$x^2+15x+36$

(d) $(2x+3)(5x+8)$

$10x^2+31x+24$

(e) $(4x-1)(x+2)$

$4x^2+7x-2$

(f) $(6x-5)(4x-3)$

$24x^2-38x+15$

3. Never forget that squaring a binomial also a process of repeated distribution. Write each of the following perfect squares as **trinomials** in **standard form**.

(a) $(x+3)^2 \neq x^2+3^2$ (b) $(x-10)^2$ $(x-10)$ (c) $(2t+3)^2$ $(2t+3)$

$(x+3)(x+3)$ $x^2-20x+100$ $4t^2+12t+9$

$x^2+\underbrace{3x+3x}+9$

x^2+6x+9

4. An interesting thing happens when you multiply two **conjugate binomials**. Conjugates have the property of having the same **terms** but differ by the operation between the two terms (in one case addition and in one case subtraction). Multiply each of the following **conjugate pairs** and state your answers in **standard form**. The first is done as an example

(a) $(x+3)(x-3)$

$= x(x-3)+3(x-3)$

$= x^2-3x+3x-9$

$= x^2-9$

(b) $(x-5)(x+5)$

$x^2 - \cancel{5x} + \cancel{5x} - 25$

$x^2 - 25$

(c) $(10+x)(10-x)$

$100 - x^2$

(d) $(2t+3)(2t-3)$

$4t^2 - 9$

(e) $(5t+1)(5t-1)$

$25t^2 - 1$

(f) $(8-3t)(8+3t)$

$64 - 9t^2$

5. Write each of the following products in standard polynomial form.

(a) $(x+3)(x-2)(x-8)$

$x^2 - 8x - 2x + 16$

$(x^2 - 10x + 16)(x+3)$

$x^3 - 10x^2 + 16x$

$3x^2 - 30x + 48$

$x^3 - 7x^2 - 14x + 48$

(b) $(x+2)(x-2)(x+3)(x-3)$ (Hint: try to use #4)

$(x^2-4)(x^2-9)$

$x^4 - 9x^2 - 4x^2 + 36$

$x^4 - 13x^2 + 36$

Reasoning

6. Notice again how similar polynomials are to integers, i.e. the set $\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$. Write a statement below for polynomials based on the statement about integers.

Statement About Integers: An integer times an integer produces an integer.

Statement About Polynomials: A polynomial times a polynomial produces a polynomial

7. Consider the product $(3x+1)^2$.

(a) Write this product in standard trinomial form.

$(3x+1)(3x+1)$

$\boxed{9x^2+6x+1}$

(b) Use your answer in part (a) to determine the value of 31^2 without your calculator.

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$9 \cdot 100$

$9 \cdot 10^2$

$9x^2$

Bell Ringer:

Choose your breath:

Initial (monitor breath's rhythm)

Heart/Belly

Calming (2 in, 4 out)

Energizing (4 in, 2 out)

Bell Ringer:

The value in dollars, v(x), of a certain car after x years is represented by the equation v(x)=25,000(0.86)^x. To the nearest dollar, how much more is the car worth after 2 years than 3 years?

a) 2589 ~~b) 6510~~ ~~c) 15901~~ ~~d) 18490~~

$t = 1$ $t = 3$ $t = 2$

$-t = 2$

Factoring expressions is one of the **gateway skills** that is necessary for much of what we do in algebra for the rest of the course. The word **factor** has two meanings and both are important.

THE TWO MEANINGS OF FACTOR

1. **Factor (verb):** To rewrite an algebraic expression as an **equivalent product**.
2. **Factor (noun):** An algebraic expression that is one part of a larger factored expression.

***Exercise* #1:** Consider the expression $6x^2+15x$.

(a) Write the individual terms $6x^2$ and $15x$ as completely factored expressions. Determine their **greatest common factor**.

$6x^2 =$ $3x(2x)$

$15x =$ $3x(5)$

(b) Using the Distributive Property, rewrite $6x^2+15x$ as a product involving the **gcf** from (a).

$3x(2x+5)$

(c) Evaluate both $6x^2+15x$ and the factored expression you wrote in (b) for $x=2$. What do you find? What does this support about the two expressions?

$6(2)^2+15(2)$

$6\cdot 4$

$24+30$

54

$3(2)(2(2)+5)$

$6(4+5)$

$6\cdot 9$

54

It is important that you are **fluent** reversing the **distributive property** in order to factor out a common factor (most often the greatest common factor). Let's get some practice in the next exercise just identifying the greatest common factors.

***Exercise* #2:** For each of the following sets of monomials, identify the greatest common factor of each. Write each term as an extended product (if necessary).

(a) $12x^3$ and $18x$

$6x$

(b) $5x^4$ and $25x^2$

$5x^2$

(c) $21x^2y^5$ and $14xy^7$

$7xy^5$

(d) $24x^3, 16x^2$, and $8x$

$8x$

(e) $20x^3, -12x^2$, and $28x$

$4x$

(f) $18x^2y^2, 45x^2y$, and $90xy^2$

$9xy$

Once you can identify the greatest common factor of a set of monomials, you can then easily use it and the distributive property to write equivalent factored expressions.

Exercise #3: Write each polynomial below as a factored expression involving the greatest common factor of the polynomial.

(a) $6x^2+10x$

$2x(3x+5)$

(b) $3x-24$

$3(x-8)$

(c) $10x^2-15x$

$5x(2x-3)$

(d) $4x^2+8x+24$

$4(x^2+2x+6)$

(e) $6x^3-8x^2+2x$

$2x(3x^2-4x+1)$

(f) $10x^3-35x^2$

$5x^2(2x-7)$

(g) $10x^2-40x-50$

$10(x^2-4x-5)$

(h) $8x^4-2x^2$

$2x^2(4x^2-1)$

(i) $8x^3+24x^2-32x$

$8x(x^2+3x-4)$

Being able to **fluently** factor out a gcf is an essential skill. Sometimes greatest common factors are more complicated than simple monomials. We have done this type of factoring back in Unit #1.

***Exercise* #4:** Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor.

(a) $\underline{(x+5)}(x-1)+\underline{(x+5)}(2x-3)$

$(x+5)(3x-4)$

(b) $\underline{(2x-1)}(2x+7)-\underline{(2x-1)}(x-3)$

$(2x-1)(x+10)$