Algebra 1A:

Unit 1-6

Multiplying and Dividing Real Numbers

Aug 24-4:27 PM

Objective To find products and quotients of real numbers

You may not know the answer, but you can make a conjecture.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

Use patterns to complete the table and answer the questions below. Explain your reasoning.

- What is the sign of the product of a positive number and a negative number?
- What is the sign of the product of two negative numbers?

$2 \cdot 3 = 6$	$-2 \cdot 3 = -6$
$2 \cdot 2 = 4$	$-2 \cdot 2 = -4$
$2 \cdot 1 = 2$	$-2 \cdot 1 = -2$
$2 \cdot 0 = \blacksquare$	$-2 \cdot 0 = \blacksquare$
$2 \cdot (-1) = \blacksquare$	$-2 \cdot (-1) = \blacksquare$
$2 \cdot (-2) = \blacksquare$	$-2 \cdot (-2) = \blacksquare$

Lesson Vocabulary
- multiplicative inverse
- reciprocal

The patterns in the Solve It suggest rules for multiplying real numbers.

Essential Understanding The rules for multiplying real numbers are related to the properties of real numbers and the definitions of operations.

You know that the product of two positive numbers is positive. For example, $3(5) = 15$. You can think about the product of a positive number and a negative number in terms of groups of numbers. For example, $3(-5)$ means 3 groups of -5. So, $3(-5) = (-5) + (-5) + (-5)$, or $3(-5) = -15$.

You can also derive the product of two negative numbers, such as $-3(-5)$.

$3(-5) = -15$	Start with the product $3(-5) = -15$.
$-[3(-5)] = -(-15)$	The opposites of two equal numbers are equal.
$-1[3(-5)] = -(-15)$	Multiplication Property of -1
$[-1(3)](-5) = -(-15)$	Associative Property of Multiplication
$-3(-5) = -(-15)$	Multiplication Property of -1
$-3(-5) = 15$	The opposite of -15 is 15.

These discussions illustrate the following rules for multiplying real numbers.

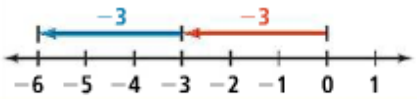
take note

Key Concept Multiplying Real Numbers

Words The product of two real numbers with *different* signs is *negative*.

Examples $2(-3) = -6$ $-2 \cdot 3 = -6$

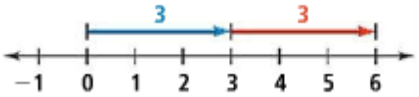
Model $2(-3) = -6$



Words The product of two real numbers with the *same* sign is *positive*.

Examples $2 \cdot 3 = 6$ $-2(-3) = 6$

Model $2 \cdot 3 = 6$



Aug 24-4:29 PM

Problem 1 Multiplying Real Numbers

Plan

What is your first step in finding a product of real numbers?
Identify the signs of the factors. Then determine the sign of the product.

What is each product?

A $12(-8) = -96$ The product of two numbers with different signs is negative.

B $24(0.5) = 12$ The product of two numbers with the same sign is positive.

C $-\frac{3}{4} \cdot \frac{1}{2} = -\frac{3}{8}$ The product of two numbers with different signs is negative.

D $(-3)^2 = (-3)(-3) = 9$ The product of two numbers with the same sign is positive.

Got It? 1. What is each product?

a. $6(-15)$ b. $12(0.2)$ c. $-\frac{7}{10}\left(\frac{3}{5}\right)$ d. $(-4)^2$

Notice that $(-3)^2 = 9$ in part (d) of Problem 1. Recall from Lesson 1-3 that a is a square root of b if $a^2 = b$. So, -3 is a square root of 9. A negative square root is represented by $-\sqrt{\ }$. Every positive real number has a positive and a negative square root. The symbol $\pm$ in front of the radical indicates both square roots.

Problem 2 Simplifying Square Root Expressions

What is the simplified form of each expression?

A $-\sqrt{25} = -5$ $(-5)^2 = 25$, so $-\sqrt{25} = -5$.

B $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$ $\left(\frac{2}{7}\right)^2 = \frac{4}{49}$ and $\left(-\frac{2}{7}\right)^2 = \frac{4}{49}$, so $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$.

Think

How can you find a negative square root?
Look for a negative number that you can multiply by itself to get a product that is equal to the radicand.

Got It? 2. What is the simplified form of each expression?

a. $\sqrt{64}$ b. $\pm\sqrt{16}$ c. $-\sqrt{121}$ d. $\pm\sqrt{\frac{1}{36}}$

8 ± 4 -11 $\pm\frac{\sqrt{1}}{\sqrt{36}} = \pm\frac{1}{6}$

Essential Understanding Rules for dividing real numbers are related to the rules for multiplying real numbers.

For any real numbers a, b, and c where $a \neq 0$, if $a \cdot b = c$, then $b = c \div a$. For instance, $-8(-2) = 16$, so $-2 = 16 \div (-8)$. Similarly $-8(2) = -16$, so $2 = -16 \div (-8)$. These examples illustrate the following rules.

take note

Key Concept Dividing Real Numbers

Words The quotient of two real numbers with *different* signs is *negative*.
Examples $-20 \div 5 = -4$ $20 \div (-5) = -4$

Words The quotient of two real numbers with the *same* sign is *positive*.
Examples $20 \div 5 = 4$ $-20 \div (-5) = 4$

Division Involving 0

Words The quotient of 0 and any nonzero real number is 0. The quotient of any real number and 0 is undefined.
Examples $0 \div 8 = 0$ $8 \div 0$ is undefined.

Aug 24-4:31 PM

Think

How is dividing similar to multiplying?
You find the sign of a quotient using the signs of the numbers you're dividing, just as you find the sign of a product using the signs of the factors.

Problem 3 Dividing Real Numbers

Sky Diving **A sky diver's elevation changes by −3600 ft in 4 min after the parachute opens. What is the average change in the sky diver's elevation each minute?**

$-3600 \div 4 = -900$ The numbers have different signs, so the quotient is negative.

The sky diver's average change in elevation is −900 ft per minute.

Got It? **3.** You make five withdrawals of equal amounts from your bank account. The total amount you withdraw is $360. What is the change in your account balance each time you make a withdrawal?

The Inverse Property of Multiplication describes the relationship between a number and its multiplicative inverse.

take note

Property Inverse Property of Multiplication

Words For every nonzero real number a, there is a multiplicative inverse $\frac{1}{a}$ such that $a(\frac{1}{a}) = 1$.

Examples The multiplicative inverse of -4 is $-\frac{1}{4}$ because $-4(-\frac{1}{4}) = 1$.

Aug 24-4:32 PM

The **reciprocal** of a nonzero real number of the form $\frac{a}{b}$ is $\frac{b}{a}$. The product of a number and its reciprocal is 1, so the reciprocal of a number is its multiplicative inverse. This suggests a rule for dividing fractions.

Here's Why It Works Let a, b, c, and d be nonzero integers.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}}$$ Write the expression as a fraction.

$$= \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}}$$ Multiply the numerator and denominator by $\frac{d}{c}$. Since this is equivalent to multiplying by 1, it does not change the quotient.

$$= \frac{\frac{a}{b} \cdot \frac{d}{c}}{1}$$ Inverse Property of Multiplication

$$= \frac{a}{b} \cdot \frac{d}{c}$$ Simplify.

This shows that dividing by a fraction is equivalent to multiplying by the reciprocal of the fraction.

Aug 24-4:32 PM

Problem 4 Dividing Fractions

Multiple Choice What is the value of $\frac{x}{y}$ when $x = -\frac{3}{4}$ and $y = -\frac{2}{3}$?

(A) $-\frac{9}{8}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{9}{8}$

$-\frac{3}{4} \div -\frac{2}{3} =$

$-\frac{3}{4} \cdot -\frac{3}{2}$

Think	Write
Rewrite the expression.	
Substitute $-\frac{3}{4}$ for x and $-\frac{2}{3}$ for y.	
Multiply by the reciprocal of $-\frac{2}{3}$.	
Simplify. Since both factors are negative, the product is positive.	$\frac{9}{8}$

The correct answer is D.

Got It? 4. a. What is the value of $\frac{3}{4} \div \left(-\frac{5}{2}\right)$?

b. **Reasoning** Is $\frac{3}{4} \div \left(-\frac{5}{2}\right)$ equivalent to $-\left(\frac{3}{4} \div \frac{5}{2}\right)$? Explain.

$\frac{3}{24} \cdot -\frac{2}{5} = -\frac{3}{10}$

Lesson Check

Do you know HOW?

Find each product. Simplify, if necessary.

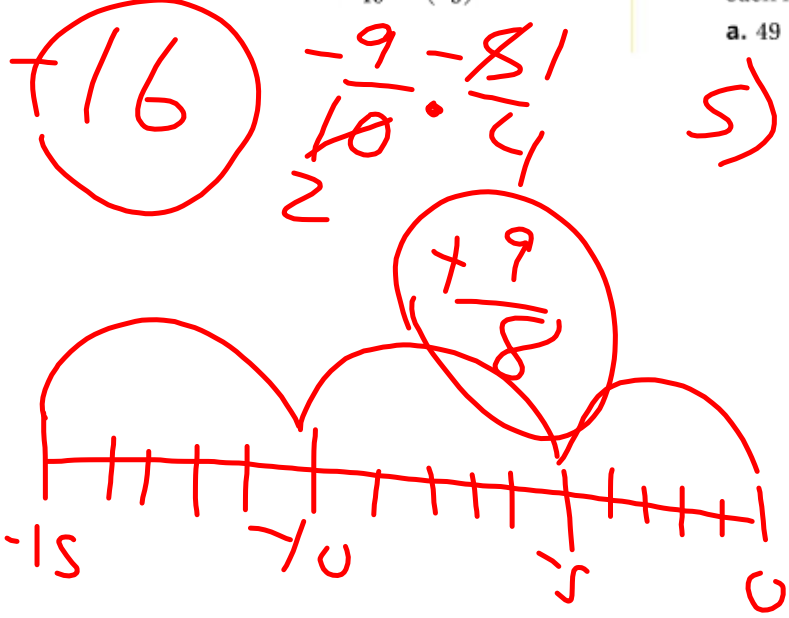
1. $-3(-12)$ 36

2. $\frac{5}{8}\left(-\frac{2}{8}\right)$ $-\frac{5}{32}$

Find each quotient. Simplify, if necessary.

3. $-48 \div 3$ -16

4. $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$ $-\frac{9}{10} \cdot -\frac{5}{4}$ $\frac{9}{8}$



Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** What is the reciprocal of $-\frac{1}{5}$?

6. **Reasoning** Use a number line to explain why $-15 \div 3 = -5$.

7. **Reasoning** Determine how many real square roots each number has. Explain your answers.

   a. 49 b. 0

5) $-\frac{1}{5}x = 1$

$x = 1 \cdot \frac{-5}{1} = -5$

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

Practice Find each product. Simplify, if necessary.

See I

8. $-8(12)$ -96
9. $8(12)$ 96
10. $7(-9)$ -63
11. $5 \cdot 4.1$ 20.5
12. $-7 \cdot 1.1$ -7.7
13. $10(-2.5)$ -25
14. $6\left(-\frac{1}{4}\right)$ -1.5
15. $-\frac{1}{9}\left(-\frac{3}{4}\right)$ $\frac{1}{12}$
16. $-\frac{3}{7} \cdot \frac{9}{10}$ $\frac{-27}{70}$
17. $-\frac{2}{11}\left(-\frac{11}{2}\right)$ 1
18. $\left(-\frac{2}{9}\right)^2$ $\frac{4}{81}$
19. $(-1.2)^2$ 1.44

Simplify each expression.

20. $\sqrt{400}$ 20

21. $\sqrt{169}$ 13

22. $-\sqrt{16}$ -4

23. $-\sqrt{900}$ -30

24. $\sqrt{\frac{36}{49}}$ $\frac{6}{7}$

25. $-\sqrt{\frac{25}{81}}$ $-\frac{5}{9}$

26. $-\sqrt{\frac{1}{9}}$ $-\frac{1}{3}$

27. $-\sqrt{\frac{121}{16}}$ $-\frac{11}{4}$

28. $\pm\sqrt{1.96}$ ± 1.4

29. $\pm\sqrt{0.25}$ ± 0.5

See Prob

Aug 24-4:34 PM

Find each quotient. Simplify, if necessary. See Pro

30. $48 \div 3$ 16

31. $-84 \div 14$ -6

32. $-39 \div (-13)$ 3

33. $\frac{63}{-21}$ -3

34. $-46 \div (-2)$ 23

35. $-8.1 \div 9$ $-.9$

36. $\frac{-121}{11}$ -11

37. $75 \div (-0.3)$ -250

Aug 24-4:35 PM

STEM **38. Scuba Diving** A scuba diver's vertical position in relation to the surface of the water changes by −90 ft in 3 min. What is the average change in the diver's vertical position each minute? -30 ft/m.

39. Part-Time Job You earn the same amount each week at your part-time job. The total amount you earn in 4 weeks is $460. How much do you earn per week?

$115 \quad \frac{460}{4} = \$115/\text{week}$

Find each quotient. Simplify, if necessary.

See P

40. $20 \div \frac{1}{4}$

$\frac{20}{\frac{1}{4}} = 20.4$

80

41. $-5 \div \left(-\frac{5}{3}\right)$

$\frac{-5}{1} \cdot \frac{-3}{5}$

3

42. $\frac{9}{10} \div \left(-\frac{4}{5}\right)$

$\frac{9}{10} \cdot \frac{-8}{4}$

$-\frac{9}{5}$

43. $-\frac{12}{13} \div \frac{12}{13}$

$\frac{-12}{13} \cdot \frac{13}{12}$

-1

Find the value of the expression $\frac{x}{y}$ for the given values of x and y. Write your answer in the simplest form.

44. $x = -\frac{2}{3}; y = -\frac{1}{4}$

$-\frac{2}{3} \cdot \frac{-4}{1}$

$\frac{8}{3}$

45. $x = -\frac{5}{6}; y = \frac{3}{5}$

$-\frac{5}{6} \cdot \frac{5}{3}$

$-\frac{25}{18}$

46. $x = \frac{2}{7}; y = -\frac{20}{21}$

$\frac{2}{7} \cdot -\frac{21}{20}$

$-\frac{3}{10}$

47. $x = \frac{3}{8}; y = \frac{3}{4}$

$\frac{3}{8} \cdot \frac{4}{3}$

$\frac{1}{2}$

Aug 24-4:36 PM

Apply

48. Think About a Plan A lumberjack cuts 7 pieces of equal length from a log, as shown at the right. What is the change in the log's length after 7 cuts? 15.75 ft

- What operation can you use to find the answer? mult.
- Will your answer be a positive value or a negative value? How do you know? -> 3 0000000

49. Farmer's Market A farmer has 120 bushels of beans for sale at a farmer's market. He sells an average of $15\frac{3}{4}$ bushels each day. After 6 days, what is the change in the total number of bushels the farmer has for sale at the farmer's market? -94.5

50. Stocks The price per share of a stock changed by −$4.50 on each of 5 consecutive days. If the starting price per share was $67.50, what was the ending price?

$45.00

$7\left(\frac{9}{4}\right)$

$\frac{63}{4}$

15.75 ft

$120 - \frac{63}{4} \cdot \frac{3}{1}$

$67.50 - 4.50(5)$

$67.50 - 22.50$

45.00

$120 - \frac{189}{2} = 120 - 94.5$

25.5 bushels

Open-Ended Write an algebraic expression that uses x, y, and z and simplifies to the given value when $x = -3$, $y = -2$, and $z = -1$. The expression should involve only multiplication or division.

51. -16

y^4z

$-2 \cdot -2 \cdot -2 \cdot -2 \cdot -1$

$16 \cdot -1 = -16$

52. 1

$z^2 = (-1)^2 =$

$-1 \cdot -1 = 1$

53. 12

xy^2z

$-3 \cdot -2 \cdot -2 \cdot -1$

$6 \cdot 2 \cdot 1$

$12 \cdot 1$

$+12$

Aug 24-4:37 PM

Evaluate each expression for $m = -5$, $n = \frac{3}{2}$, and $p = -8$.

54. $-7m - 10n$ (-5) $(\frac{3}{2})$ $35 - 15 = 20$

55. $-3mnp$ -180

56. $8n \div (-6p)$

57. $2p^2(-n) \div m$

58. Look for a Pattern Extend the pattern in the diagram to six factors of -2. What rule describes the sign of the product based on the number of negative factors? even = +, odd = −

-2(-2) = 4
-2(-2)(-2) = -8
-2(-2)(-2)(-2) = 16

STEM **59. Temperature** The formula $F = \frac{9}{5}C + 32$ changes a temperature reading from the Celsius scale C to the Fahrenheit scale F. What is the temperature measured in degrees Fahrenheit when the Celsius temperature is $-25°C$?

$F = -45 + 32 = -13$