

Algebra 1A

Unit 1-5:

Adding and Subtracting with
Real Numbers

Objective To find sums and differences of real numbers



You may find using a number line helpful here.



Getting Ready!

You have kept track of the activity on a gift card, as shown at the right. The values are negative (red) when you spend money and positive (black) when you add money.

You want to give the card to a friend. How much money must you add to make the card worth \$25? Explain your reasoning.

9/3 get gift card	\$50
9/4 buy new game	\$19
9/7 buy new jacket	\$29
9/10 Aunt Sue adds \$	\$25
9/13 buy new headphones	\$13
need to add to be \$25	?

$$\begin{array}{r} 50 \\ -48 \\ \hline 2 + 25 = 27 \\ -13 \\ \hline 14 + 11 = 25 \end{array}$$

$$\begin{array}{r} 14 + x = 25 \\ -14 \\ \hline x = 11 \end{array}$$

Lesson Vocabulary

- absolute value
- opposites
- additive inverses

Essential Understanding You can add or subtract any real numbers using a number line model. You can also add or subtract real numbers using rules involving absolute value.

Think

How do you know which direction to move along the number line?
 If the number added is positive, move to the right. If the number added is negative, move to the left.

Problem 1 Using Number Line Models

What is each sum? Use a number line.

A $3 + 5$

Start at 3. Move 5 units right. $3 + 5 = 8$

B $3 + (-5)$

move 5 units left. $3 + (-5) = -2$

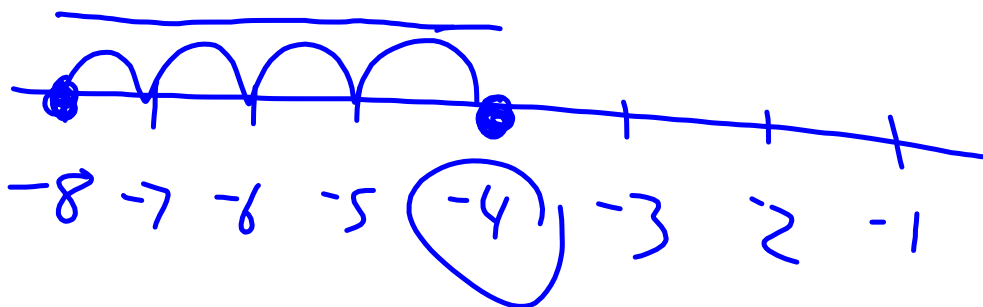
C $-3 + 5$

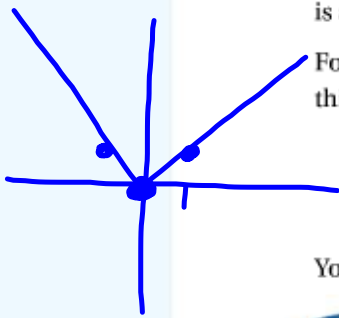
move 5 units right. $-3 + 5 = 2$

D $-3 + (-5)$

move 5 units left. $-3 + (-5) = -8$

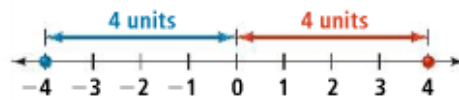
Got It? 1. What is $-8 + 4$? Use a number line.





The **absolute value** of a number is its distance from 0 on a number line. Absolute value is always nonnegative since distance is always nonnegative.

For example, the absolute value of 4 is 4 and the absolute value of -4 is 4. You can write this as $|4| = 4$ and $|-4| = 4$.



You can use absolute value when you find the sums of real numbers.

take note

Key Concept Adding Real Numbers

Dynamic Activity Adding Real Numbers

Adding Numbers With the Same Sign

To add two numbers with the same sign, add their absolute values. The sum has the same sign as the addends.

Examples $3 + 4 = 7$ $-3 + (-4) = -7$

Adding Numbers With Different Signs

To add two numbers with different signs, subtract their absolute values. The sum has the same sign as the addend with the greater absolute value.

Examples $-3 + 4 = 1$ $3 + (-4) = -1$

Plan

What is the first step in finding each sum?

Identify whether the addends have the same sign or different signs. Then choose the appropriate rule to use.

© Problem 2 Adding Real Numbers

What is each sum?

A $-12 + 7$

$-12 + 7 = -5$

The difference of the absolute values is 5. The negative addend has the greater absolute value. The sum is negative.

B $-18 + (-2)$

$-18 + (-2) = -20$

The addends have the same sign (negative), so add their absolute values. The sum is negative.

C $-4.8 + 9.5$

$9.5 - 4.8 = 4.7$

D $\frac{3}{4} + (-\frac{5}{6})$

$\frac{9}{12} + \frac{-10}{12} = \frac{10-9}{12} = -\frac{1}{12}$

Got It? 2. What is each sum?

a. $-16 + (-8)$

-24

b. $-11 + 9$

$11-9 = 11-9$
 -2

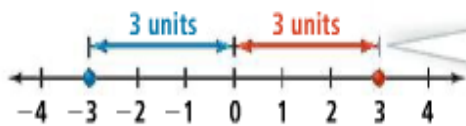
c. $9 + (-11)$

-2

d. $-6 + (-2)$

-8

Two numbers that are the same distance from 0 on a number line but lie in opposite directions are **opposites**.



-3 and 3 are the same distance from 0. So -3 and 3 are opposites.

A number and its opposite are called **additive inverses**. To find the sum of a number and its opposite, you can use the **Inverse Property of Addition**.

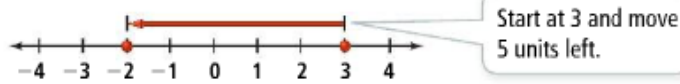
Take note

Property Inverse Property of Addition

For every real number a , there is an additive inverse $-a$ such that
 $a + (-a) = -a + a = 0$.

Examples $14 + (-14) = 0$ $-14 + 14 = 0$

You can use opposites (additive inverses) to subtract real numbers. To see how, look at the number line below, which models $3 - 5$ and $3 + (-5)$.



$3 - 5$ and $3 + (-5)$ are equivalent expressions, illustrating the rule below.



Key Concept Subtracting Real Numbers

To subtract a real number, add its opposite: $a - b = a + (-b)$.

Examples $3 - 5 = 3 + (-5) = -2$ $3 - (-5) = 3 + 5 = 8$

Think

Why rewrite subtraction as addition?

You can simplify expressions using the rules for adding real numbers that you learned earlier in this lesson.



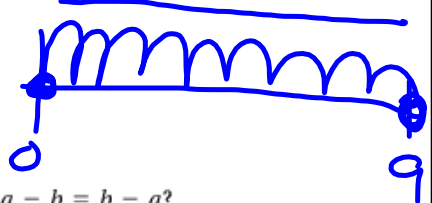
Problem 3 Subtracting Real Numbers

What is each difference?

A $-8 - (-13) = -8 + 13$ The opposite of -13 is 13 . So add 13 .
 $= 5$ Use rules for addition.

B $3.5 - 12.4 = 3.5 + (-12.4)$ The opposite of 12.4 is -12.4 . So add -12.4 .
 $= -8.9$ Use rules for addition.

C $9 - 9 =$ $9 + -9$



Got It? 3. a. What is $4.8 - (-8.7)$?
 b. **Reasoning** For what values of a and b does $a - b = b - a$?

13.5

~~$\frac{1}{2} = \frac{1}{4} = \frac{1}{4} - \frac{1}{2}$~~
 ~~$1 + 2 = -2 + 1$~~
 ~~$-3 = -3$~~
 ~~$-3 = 1 + 2$~~

All of the addition properties of real numbers that you learned in Lesson 1-4 apply to both positive and negative numbers. You can use these properties to reorder and simplify expressions.

Problem 4 Adding and Subtracting Real Numbers

Scuba Diving A reef explorer dives 25 ft to photograph brain coral and then rises 16 ft to travel over a ridge before diving 47 ft to survey the base of the reef. Then the diver rises 29 ft to see an underwater cavern. What is the location of the cavern in relation to sea level?

Think

How do you represent the problem with an expression?
Start your expression with zero to represent sea level. Subtract for dives, and add for rises.

Know

Distance and direction for each change in location

Need

Location in relation to sea level after changes

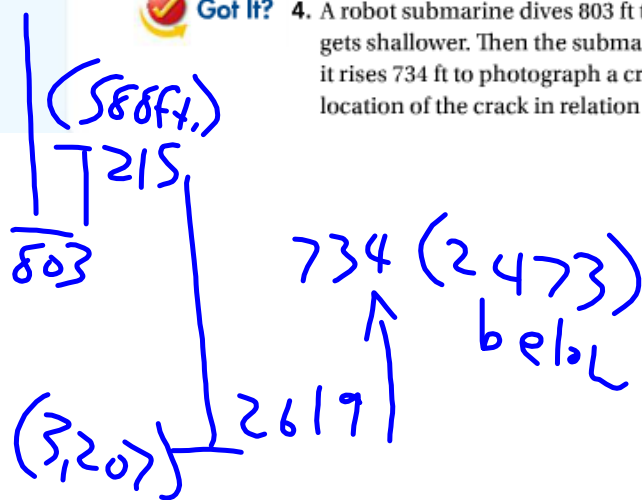
Plan

Represent the diver's trip with an expression. Reorder the values to make calculations easier.

$0 - 25 + 16 - 47 + 29$	Write an expression.
$= 0 + (-25) + 16 + (-47) + 29$	Use rule for subtracting real numbers.
$= 0 + 16 + 29 + (-25) + (-47)$	Commutative Property of Addition
$= 0 + (16 + 29) + [(-25) + (-47)]$	Group addends with the same sign.
$= 0 + 45 + (-72)$	Add inside grouping symbols.
$= 45 + (-72)$	Identity Property of Addition
$= -27$	Use rule for adding numbers with different signs.

The cavern is at -27 ft in relation to sea level.

Got It? 4. A robot submarine dives 803 ft to the ocean floor. It rises 215 ft as the water gets shallower. Then the submarine dives 2619 ft into a deep crevice. Next, it rises 734 ft to photograph a crack in the wall of the crevice. What is the location of the crack in relation to sea level?



Lesson Check

Do you know HOW?

Use a number line to find each sum.

1. $-5 + 2$ -3 2. $-2 + (-1)$ -3

Find each sum or difference.

3. $-12 + 9$ -3 4. $-4 + (-3)$ -7

5. $-3 + (+5)$ 2 6. $1.5 - 8.5$ -7

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** What is the sum of a number and its opposite? 0 ; Add. Inv.
8. **Compare and Contrast** How is subtraction related to addition?
9. **Error Analysis** Your friend says that since $-a$ is the opposite of a , the opposite of a number is always negative. Describe and correct the error.

Practice and Problem-Solving Exercises**Practice**

Use a number line to find each sum.

See Pr

10. $2 + 5$

11. $-3 + 8$

12. $4 + (-3)$

13. $1 + (-6)$

14. $-6 + 9$

15. $-4 + 7$

16. $-6 + (-8)$

17. $-9 + (-3)$

Find each sum.

 See Pro

18. $11 + 9$

19. $17 + (-28)$

20. $12 + (-9)$

21. $-2 + 7$

22. $-14 + (-10)$

23. $-9 + (-2)$

24. $3.2 + 1.4$

25. $5.1 + (-0.7)$

26. $-2.2 + (-3.8)$

27. $\frac{1}{2} + \left(-\frac{7}{2}\right)$

28. $-\frac{2}{3} + \left(-\frac{3}{5}\right)$

29. $\frac{7}{9} + \left(-\frac{5}{12}\right)$

Find each difference.

 See Pr

30. $5 - 15$

31. $-13 - 7$

32. $-19 - 7$

33. $36 - (-12)$

34. $-29 - (-11)$

35. $-7 - (-5)$

36. $8.5 - 7.6$

37. $-2.5 - 17.8$

38. $-2.9 - (-7.5)$

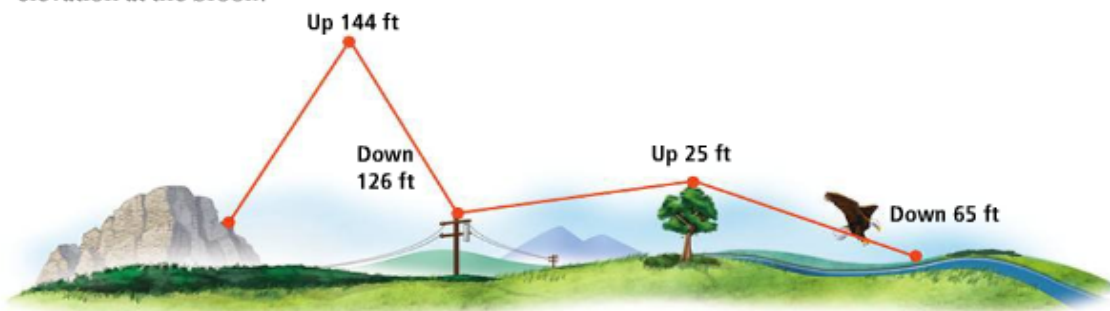
39. $3.5 - 1.9$

40. $\frac{1}{8} - \frac{3}{4}$

41. $\frac{7}{16} - \left(-\frac{1}{2}\right)$

42. **Bird Watching** An eagle starts flying at an elevation of 42 ft. Elevation is the distance above sea level. The diagram below shows the elevation changes during the eagle's flight. Write an expression representing the eagle's flight. What is the elevation at the brook?

➡ See Problem 4



43. **Stock Market** A stock's starting price per share is \$51.47 at the beginning of the week. During the week, the price changes by gaining \$1.22, then losing \$3.47, then losing \$2.11, then losing \$.98, and finally gaining \$2.41. What is the ending stock price?

Evaluate each expression for $a = -2$, $b = -4.1$, and $c = 5$.

44. $a - b + c$

45. $-c + b - a$

46. $-a + (-c)$

- © 47. **Error Analysis** Describe and correct the error in finding the difference shown at the right.

$$\begin{aligned}
 -4 - (-5) &= -4 + (-(-5)) \\
 &= -4 + 5 \\
 &= -1
 \end{aligned}$$

- © 48. **Writing** Without calculating, tell which is greater, the sum of -135 and 257 or the sum of 135 and -257 . Explain your reasoning.

Simplify each expression.

49. $1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$

50. $7 + (2^2 - 3^2)$
 $4 - 9$

51. $-2.1 - [2.3 + (3.5 + (+1.9))]$

$7 - 5 = 2$

$1 + 7.1 = 8.1$

52. **Think About a Plan** In golf, the expected number of strokes is called "par." When the number of strokes taken is more than par, your score is positive. When the number of strokes is less than par, your score is negative. The lowest score wins.

$-\frac{1}{12}$

The scorecard shows par and one golfer's score for the first four holes played on a nine-hole golf course. The golfer's scores on the remaining five holes are $-1, 0, -1, +1, 0$. Par for the nine holes is 36. What is the golfer's total number of strokes for the nine holes?

Golf Scorecard

Par	Number of Strokes	Score
4	6	+2
4	3	-1
3	3	0
5	3	-2

- Can you solve the problem by adding the strokes taken on each hole?
- How is the sum of the golfer's scores related to the total number of strokes taken?

© Reasoning Use reasoning to determine whether the value of each expression is *positive* or *negative*. Do not calculate the exact answers.

53. $-225 + 318$

54. $-\frac{7}{8} + \frac{1}{3}$

55. $34.5 + 12.9 - 50$

STEM 56. **Temperature Scales** The Kelvin temperature scale is related to the degrees Celsius ($^{\circ}\text{C}$) temperature scale by the formula $x = 273 + y$, where x is the number of kelvins and y is the temperature in degrees Celsius. What is each temperature in kelvins?

a. -22°C

b. 0°C

c. -32°C

© Writing 57. Explain how you can tell without calculating whether the sum of a positive number and a negative number will be positive, negative, or zero.

Decide whether each statement is true or false. Explain your reasoning.

58. The sum of a positive ~~number~~^F and a negative number is always negative.

59. The difference of two numbers is always less than the sum of those two numbers. ?!

60. A number minus its ~~opposite~~^T is twice the number.

- STEM 61. Meteorology** Weather forecasters use a barometer to measure air pressure and make weather predictions. Suppose a standard mercury barometer reads 29.8 in. The mercury rises 0.02 in. and then falls 0.09 in. The mercury falls again 0.18 in. before rising 0.07 in. What is the final reading on the barometer?
- 62. Multiple Choice** Which expression is equivalent to $x - y$?
- (A) $y - x$ (B) $x - (-y)$ (C) $x + (-y)$ (D) $y + (-x)$
- STEM 63. Chemistry** Atoms contain particles called protons and electrons. Each proton has a charge of $+1$ and each electron has a charge of -1 . A certain sulfur ion has 18 electrons and 16 protons. The charge on an ion is the sum of the charges of its protons and electrons. What is the sulfur ion's charge?

Challenge

64. Reasoning If $|x| > |y|$, does $|x - y| = |x| - |y|$? Justify your answer.



65. Reasoning A student wrote the equation $-|m| = |-m|$. Is the equation *always*, *sometimes*, or *never* true? Explain.

Simplify each expression.

$|5| > |2|$

66. $\frac{c}{4} - \frac{c}{4} = 0$

67. $\frac{w}{5} + \left(-\frac{w}{10}\right) = \frac{w}{10}$

68. $\frac{d}{5} - \left(-\frac{d}{5}\right) = \frac{2d}{5}$

69. Reasoning Answer each question. Justify your answers.

a. Is $|a - b|$ always equal to $|b - a|$?

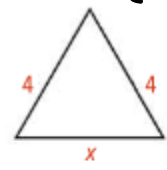
b. Is $|a + b|$ always equal to $|a| + |b|$?

$|5-2| = 5-2$
 $3 = 3 \checkmark$

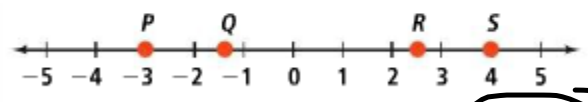
$| -5 | > | 2 |$
 $5 \quad 2$
 $| -5+2 | = 5-2$
 $1-2$
 $7 = 3 \times$

Standardized Test Prep

70. What is the value of $-b - a$ when $a = -4$ and $b = 7$? $-7 + 4$
- (A) -11 (B) -3 (C) 3 (D) 11
71. Which expression is equivalent to $19 - 41$? $= -22$
- (F) $|19 - 41|$ (G) $|19 + 41|$ (H) $-|19 - 41|$ (I) $-|19 + 41|$
72. Which equation illustrates the Identity Property of Multiplication?
- (A) $x \cdot 0 = 0$ (B) $x \cdot 1 = x$ (C) $x(yz) = (xy)z$ (D) $x \cdot y = y \cdot x$
73. What is an algebraic expression for the perimeter of the triangle?
- (E) $8 + x$ (H) 8 (G) $4x$ (I) $4 + x$



74. Which point on the number line below is the best estimate for $\sqrt{8}$?



- (A) P (B) Q (C) R (D) S

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \sqrt{2} = 2\sqrt{2}$$