Algebra 1A

Unit 1-4:

Properties of Real Numbers

Aug 24-3:50 PM

Objective To identify and use properties of real numbers

SOLVE IT!

Getting Ready!

Tell whether each pair of expressions is equal by completing each statement with = or ≠. Explain your answers.

$34 + 12 \underline{?} 12 + 34$

$100 - 1 \underline{?} 1 - 100$

$0 + 180 \underline{?} 180$

$18 \div \frac{1}{18} \underline{?} 1$

$45 - 1 \underline{?} 45$

$6 \times \frac{1}{6} \underline{?} 1$

MATHEMATICAL PRACTICES The Solve It illustrates numerical relationships that are always true for real numbers.

Lesson Vocabulary
- equivalent expressions
- deductive reasoning
- counterexample

Essential Understanding Relationships that are always true for real numbers are called *properties*, which are rules used to rewrite and compare expressions.

Two algebraic expressions are **equivalent expressions** if they have the same value for all values of the variable(s). The following properties show expressions that are equivalent for all real numbers.

take note

Properties Properties of Real Numbers

Let a, b, and c be any real numbers.

Commutative Properties of Addition and Multiplication

Changing the order of the addends does not change the sum. Changing the order of the factors does not change the product.

	Algebra	Example
Addition	$a + b = b + a$	$18 + 54 = 54 + 18$
Multiplication	$a \cdot b = b \cdot a$	$12 \cdot \frac{1}{2} = \frac{1}{2} \cdot 12$

Associative Properties of Addition and Multiplication

Changing the grouping of the addends does not change the sum. Changing the grouping of the factors does not change the product.

Addition	$(a + b) + c = a + (b + c)$	$(23 + 9) + 4 = 23 + (9 + 4)$
Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(7 \cdot 9) \cdot 10 = 7 \cdot (9 \cdot 10)$

take note

Properties — Properties of Real Numbers

Let a be any real number.

Identity Properties of Addition and Multiplication

The sum of any real number and 0 is the original number. The product of any real number and 1 is the original number.

	Algebra	Example
Addition	$a + 0 = a$	$5\frac{3}{4} + 0 = 5\frac{3}{4}$
Multiplication	$a \cdot 1 = a$	$67 \cdot 1 = 67$

Zero Property of Multiplication

The product of a and 0 is 0.	$a \cdot 0 = 0$	$18 \cdot 0 = 0$

Multiplication Property of −1

The product of −1 and a is $-a$.	$-1 \cdot a = -a$	$-1 \cdot 9 = -9$

Think

What math symbols give you clues about the properties?
Parentheses, operation symbols, and the numbers 0 and 1 may indicate certain properties.

Problem 1 Identifying Properties

What property is illustrated by each statement?

A $42 \cdot 0 = 0$ Zero Property of Multiplication

B $(y + 2.5) + 28 = y + (2.5 + 28)$ Associative Property of Addition

C $10x + 0 = 10x$ Identity Property of Addition

Got It? **1.** What property is illustrated by each statement?

a. $4x \cdot 1 = 4x$

Id, ×

b. $x + (\sqrt{y} + z) = x + (z + \sqrt{y})$

C.P., +

You can use properties to help you solve some problems using mental math.

Problem 2 Using Properties for Mental Calculations

Plan

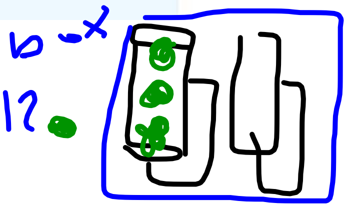
How can you make the addition easier? Look for numbers having decimal parts you can add easily, such as 0.75 and 0.25.

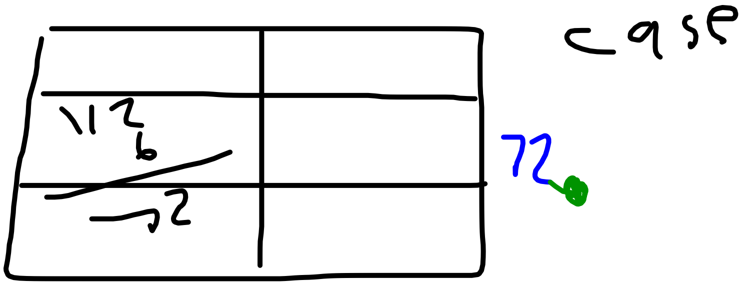
Movies **A movie ticket costs \$7.75. A drink costs \$2.40. Popcorn costs \$1.25. What is the total cost for a ticket, a drink, and popcorn? Use mental math.**

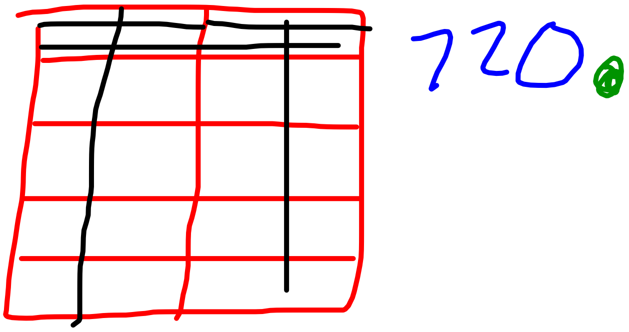
$(7.75 + 2.40) + 1.25$	$= (2.40 + 7.75) + 1.25$	Commutative Property of Addition
	$= 2.40 + (7.75 + 1.25)$	Associative Property of Addition
	$= 2.40 + 9$	Simplify inside parentheses.
	$= 11.40$	Add.

The total cost is \$11.40.

Got It? **2.** A can holds 3 tennis balls. A box holds 4 cans. A case holds 6 boxes. How many tennis balls are in 10 cases? Use mental math.







Problem 3 Writing Equivalent Expressions

Simplify each expression.

A $5(3n)$

$(5\cdot 3)\cdot n$

$15\cdot n$

$15n$

Know	Need	Plan
An expression	Groups of numbers that can be simplified	Use properties to group or reorder parts of the expression.

$5(3n) = (5\cdot 3)n$ Associative Property of Multiplication

$= 15n$ Simplify.

$\frac{y}{y}\cdot\frac{1}{1} = 1\ \ y^0$

B $(4 + 7b) + 8$

$(4 + 7b) + 8 = (7b + 4) + 8$ Commutative Property of Addition

$= 7b + (4 + 8)$ Associative Property of Addition

$= 7b + 12$ Simplify.

C $\frac{6xy}{y}$

$\frac{6\cdot x\cdot y}{y} = \frac{(6x)\cdot y}{y} = \frac{6x}{1}\cdot\frac{y}{y}$ ①

$\frac{6xy}{y} = \frac{6x\cdot y}{1\cdot y}$ Rewrite denominator using Identity Property of Multiplication.

$= \frac{6x}{1}\cdot\frac{y}{y}$ Use rule for multiplying fractions: $\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}$.

$= 6x\cdot 1$ $x \div 1 = x$ and $y \div y = 1$.

$= 6x$ Identity Property of Multiplication

Got It? 3. Simplify each expression.

a. $2.1(4.5x)$

$(2.1\cdot 4.5)x$

$9.45x$

b. $6 + (4h + 3)$

$9 + 4h$

c. $\frac{8m}{12mn}$

$\frac{8}{12}\ \frac{m}{m}\ \frac{1}{n}$

$\frac{8\cdot 1\cdot 1}{12\cdot 1\cdot n} = \frac{8}{12n}$

$\frac{2}{3}n$

In Problem 3, reasoning and properties were used to show that two expressions are equivalent. This is an example of *deductive reasoning*. **Deductive reasoning** is the process of reasoning logically from given facts to a conclusion.

To show that a statement is *not* true, find an example for which it is not true. An example showing that a statement is false is a **counterexample**. You need only one counterexample to prove that a statement is false.

Problem 4 Using Deductive Reasoning and Counterexamples

Is the statement *true* or *false*? If it is false, give a counterexample.

A **For all real numbers a and b, $a \cdot b = b + a$.**

False. $5 \cdot 3 \neq 3 + 5$ is a counterexample.

B **For all real numbers a, b, and c, $(a + b) + c = b + (a + c)$.**

True. Use properties of real numbers to show that the expressions are equivalent.

$(a + b) + c = (b + a) + c$ Commutative Property of Addition

$= b + (a + c)$ Associative Property of Addition

Plan

Look for a counterexample to show the statement is false. If you don't find one, try to use properties to show that it is true.

Got It? 4. **Reasoning** **Is each statement in parts (a) and (b) *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.**

T **a.** For all real numbers *j* and *k*, $j \cdot k = (k + 0) \cdot j$.

F **b.** For all real numbers *m* and *n*, $m(n + 1) = mn + 1$.

c. Is the statement in part (A) of Problem 4 false for *every* pair of real numbers *a* and *b*? Explain.

$m(n+1)$

$mn + m \neq mn + 1$

Lesson Check

Do you know HOW?

Name the property that each statement illustrates.

1. $x + 12 = 12 + x$ C.P., +
2. $5 \cdot (12 \cdot x) = (5 \cdot 12) \cdot x$ a.p., +
3. You buy a sandwich for \$2.95, an apple for \$.45, and a bottle of juice for \$1.05. What is the total cost? 4.45
4. Simplify $\frac{24cd}{c}$.

$\frac{24 \cdot d \cdot \cancel{c}^{1}}{\cancel{c}_{1}} = 24 \cdot d \cdot 1 = 24d$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Tell whether the expressions in each pair are equivalent.
   a. $5x \cdot 1$ and $1 + 5x$ ✗
   b. $1 + (2t + 1)$ and $2 + 2t$ ✓
6. Justify each step.

$3 \cdot (10 \cdot 12) = 3 \cdot (12 \cdot 10)$ C.P., mult.
$= (3 \cdot 12) \cdot 10$ a.p., mult.
$= 36 \cdot 10$ simplify
$= 360$