

Algebra 1A:  
Unit 1-2 Order of Operations and  
Evaluating Expressions

**Objectives** To simplify expressions involving exponents  
To use the order of operations to evaluate expressions

**SOLVE IT!** **Getting Ready!**

You've won! For a door prize, you get to choose between the two options shown. Which is the better prize? Why?

**PRIZE 1**  
You get \$60 immediately.

**PRIZE 2**  
You get \$1 the first day.  
Then, each day for the next five days, you get twice the previous day's amount.

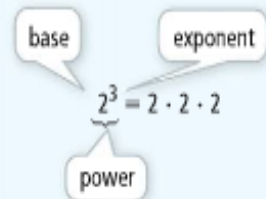
**Dynamic Activity**  
Order of Operations

- Lesson Vocabulary**
- power
  - exponent
  - base
  - simplify
  - evaluate

**Essential Understanding** You can use *powers* to shorten how you represent repeated multiplication, such as  $2 \times 2 \times 2 \times 2 \times 2 \times 2$ .

A **power** has two parts, a *base* and an *exponent*. The **exponent** tells you how many times to use the **base** as a factor. You read the power  $2^3$  as "two to the third power" or "two cubed." You read  $5^2$  as "five to the second power" or "five squared."

You **simplify** a numerical expression when you replace it with its single numerical value. For example, the simplest form of  $2 \cdot 8$  is 16. To simplify a power, you replace it with its simplest name.



**Problem 1 Simplifying Powers**

What is the simplified form of the expression?

**A**  $10^7$

**B**  $(0.2)^5$

**Got It?** 1. What is the simplified form of each expression?

a.  $3^4$

$$3^2 \cdot 3^2 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

b.  $(\frac{2}{3})^3$

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

c.  $(0.5)^3$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = 0.125$$

**Essential Understanding** When simplifying an expression, you need to perform operations in the correct order.

You might think about simplifying the expression  $2 + 3 \times 5$  in two ways:

Add first.

$$2 + 3 \times 5 = 5 \times 5 = 25 \quad \times$$

Multiply first.

$$2 + 3 \times 5 = 2 + 15 = 17 \quad \checkmark$$

Both results may seem sensible, but only the second result is considered correct. This is because the second way uses the order of operations that mathematicians have agreed to follow. Always use the following order of operations:

Take note

### Key Concept Order of Operations

1. Perform any operation(s) inside grouping symbols, such as parentheses ( ) and brackets [ ]. A fraction bar also acts as a grouping symbol.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.



### Problem 2 Simplifying a Numerical Expression

What is the simplified form of each expression?

**A**  $(6 - 2)^3 \div 2$

$$(4)^3 \div 2 = 64 \div 2 = 32$$

**B**  $\frac{2^4 - 1}{5}$

$$\frac{2 \cdot 2 \cdot 2 \cdot 2 - 1}{5} = \frac{16 - 1}{5} = \frac{15}{5} = 3$$



**Got It?** 2. What is the simplified form of each expression?

a.  $5 \cdot 7 - 4^2 \div 2$

b.  $12 - 25 \div 5$

c.  $\frac{4 + 3^4}{7 - 2}$

d. **Reasoning** How does a fraction bar act as a grouping symbol? Explain.

$$\frac{4 + 81}{5} = \frac{85}{5} = 17$$

When two or more variables, or a number and variables, are written together, treat them as if they were within parentheses. So  $4xy$  is equivalent to  $(4xy)$ , and  $xy^2 = (xy^2)$ . You **evaluate** an algebraic expression by replacing each variable with a given number. Then simplify the expression using the order of operations.

**Plan**

**How is this Problem like ones you've seen before?**

You begin by substituting numbers for the variables. After substituting, you have numerical expressions just like the ones in Problem 2.

**Problem 3 Evaluating Algebraic Expressions**

What is the value of the expression for  $x = 5$  and  $y = 2$ ?

**A**  $x^2 + x - 12 \div y^2$

$$5^2 + 5 - 12 \div 2^2$$

$$25 + 5 - 12 \div 4$$

$$25 + 5 - 3$$

$$30 - 3 = 27$$

**B**  $(xy)^2 \div xy$

$$\frac{(xy)^2}{xy} = \frac{(xy)(xy)}{xy} = \frac{x^2 y^2}{xy} = x^{2-1} y^{2-1} = xy$$

**Got It?** 3. What is the value of each expression when  $a = 3$  and  $b = 4$  in parts (a)-(b)?

a.  $3b - a^2$

b.  $2b^2 - 7a$

$$3(4) - 3^2$$

$$12 - 9$$

$$3$$

$$2(4)^2 - 7(3)$$

$$2 \cdot 16 - 21$$

$$32 - 21$$

$$11$$

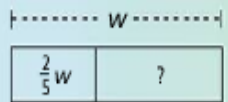
**Problem 4 Evaluating a Real-World Expression**

**Banking** What is an expression for the spending money you have left after depositing  $\frac{2}{5}$  of your wages in savings? Evaluate the expression for weekly wages of \$40, \$50, \$75, and \$100.

**Think**

How can a model help you write the expression?

This model shows that spending money equals your wages  $w$  minus the amount you save:  $\frac{2}{5}w$ .



**Know**

- Savings equals  $\frac{2}{5}$  of wages.
- Various weekly wages

**Need**

- Expression for spending money
- Amount of spending money for various weekly wages

**Plan**

Write an algebraic expression and evaluate it for each amount of weekly wages. Use a table to organize your results.

**Relate** spending money equals wages minus  $\frac{2}{5}$  of wages

**Define** Let  $w$  = your wages.

**Write**  $w - \frac{2}{5} \cdot w$

The expression  $w - \frac{2}{5} \cdot w$  represents the amount of money you have left after depositing  $\frac{2}{5}$  of your wages in savings.

**Spending Money**

Wages ( $w$ )	$w - \frac{2}{5}w$	Total Spending Money (\$)
40	$40 - \frac{2}{5}(40)$	24
50	$50 - \frac{2}{5}(50)$	30
75	$75 - \frac{2}{5}(75)$	45
100	$100 - \frac{2}{5}(100)$	60



Got It?

4. The shipping cost for an order at an online store is  $\frac{1}{10}$  the cost of the items you order. What is an expression for the total cost of a given order? What are the total costs for orders of \$43, \$79, \$95, and \$103?

$$C + \frac{1}{10} \cdot C$$

### Lesson Check

#### Do you know HOW?

What is the simplified form of each expression?

1.  $5^2 = 25$     2.  $2^3 = 8$     3.  $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Evaluate each expression for  $x = 3$  and  $y = 4$ .

4.  $x^2 + 2(x + y)$      $9 + 2(3 + 4)$

5.  $(xy)^3$      $1728$      $2^3$

6.  $4x^2 - 3xy$   
 $36 - 36 = 0$

#### Do you UNDERSTAND?



7. **Vocabulary** Identify the exponent and the base in  $4^3$ .

8. **Error Analysis** A student simplifies an expression as shown below. Find the error and simplify the expression correctly.

$$\begin{aligned}
 23 - 8 \cdot 2 + 3^2 &= 23 - 8 \cdot 2 + 9 \\
 &= 15 \cdot 2 + 9 \\
 &= 30 + 9 \\
 &= 39 \quad \times
 \end{aligned}$$

$23 - 16 + 9$   
 $7 + 9$   
 $16$





## Practice and Problem-Solving Exercises



Practice

Simplify each expression.

See Problems 1 and 2.

9.  $3^5$

10.  $4^3$

11.  $2^4$

12.  $10^8$

13.  $\left(\frac{2}{3}\right)^3$

14.  $\left(\frac{1}{2}\right)^4$

15.  $(0.4)^6$

16.  $7^4$

17.  $20 - 2 \cdot 3^2$

18.  $6 + 4 \div 2 + 3$

19.  $(6^2 - 3^3) \div 2$

20.  $5 \cdot 2^2 \div 2 + 8$

21.  $80 - (4 - 1)^3$

22.  $52 + 8^2 - 3(4 - 2)^3$

23.  $\frac{6^4 \div 3^2}{9}$

24.  $\frac{2 \cdot 7 + 4}{9 \div 3}$

Evaluate each expression for  $s = 4$  and  $t = 8$ .

25.  $(s + t)^3$

26.  $s^4 + t^2 + s \div 2$

27.  $(st)^2 \div st^2$

28.  $3st^2 \div st + 6$

29.  $(t - s)^5$

30.  $(2s)^2t$

31.  $2st^2 - s^2$

32.  $2s^2 - t^3 \div 16$

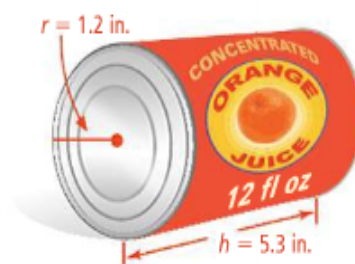
33.  $\frac{(3s)^3t + t}{s}$

- 34.** Write an expression for the amount of change you will get when you pay for a purchase  $p$  with a \$20 bill. Make a table to find the amounts of change you will get for purchases of \$11.59, \$17.50, \$19.00, and \$20.00.
- 35.** An object's momentum is defined as the product of its mass  $m$  and velocity  $v$ . Write an expression for the momentum of an object. Make a table to find the momentums of a vehicle with a mass of 1000 kg moving at a velocity of 15 m/s, 20 m/s, and 25 m/s.

**B Apply**

© 36. **Geometry** The expression  $\pi r^2 h$  represents the volume of a cylinder with radius  $r$  and height  $h$ .

- a. What is the volume, to the nearest tenth of a cubic inch, of the juice can at the right? Use 3.14 for  $\pi$ .
- b. **Reasoning** About how many cubic inches, to the nearest tenth of a cubic inch, does a fluid ounce of juice fill?



Simplify each expression.

37.  $2[(8 - 4)^5 \div 8]$

38.  $3[(4 - 2)^5 - 20]$

39.  $10 - (2^3 + 4) \div 3 - 1$

40.  $\frac{22 + \cancel{15} + (\cancel{5} - \cancel{3})}{2^3}$

41.  $3[42 - 2(10^2 - 9^2)]$

42.  $\frac{2[8 + (67 - 2^6)^3]}{9}$

37)  $2[4^5 \div 8]$

$2[1024 \div 8]$

$2[128]$

$256$

40)  $\frac{22 + 1 + 32}{8}$

$\frac{55}{8}$

42)  $2 \frac{[8 + (67 - 64)^3]}{9}$

$\frac{2(8 + 3^3)}{9} = \frac{2(8 + 27)}{9} = \frac{2(35)}{9} =$

$\frac{70}{9}$

- © 43. **Think About a Plan** The snack bar at your school has added sushi to its menu. The ingredients for one roll include sushi rice, seaweed sheets, cucumbers, cream cheese, and 3 oz of smoked salmon. One roll can be cut into 8 servings. Write an expression for the amount of salmon needed to make  $s$  servings of sushi. How much salmon is needed to make 16 servings? 24 servings? 80 servings? 100 servings?
- What operations are needed in your calculations?
  - Use a table to help you organize your results. What will you use for the column headings in your table?

44. **Salary** You earn \$10 for each hour you work at a canoe rental shop. Write an expression for your salary for working the number of hours  $h$ . ~~Make a table to find how much you earn for working 10 h, 20 h, 30 h, and 40 h.~~

$$10 \cdot h \text{ hrs} =$$
$$10h \text{ or } 10x$$

Evaluate each expression for the given values of the variables.

45.  $3(s - t)^2$ ;  $s = 4, t = 1$

46.  $2x - y^2$ ;  $x = 7, y = 3.5$

47.  $3m^2 - n$ ;  $m = 2, n = 6$

48.  $(2a + 2b)^2$ ;  $a = 3, b = 4$

49.  $2p^2 + (2q)^2; p = 4, q = 3$

51.  $\frac{3g + 6}{h}; g = 5, h = 7$

50.  $(4c - d + 0.2)^2 - 10c; c = 3.1, d = 4.6$

52.  $\frac{2w + 3v}{v^2}; v = 6, w = 1$



53. **Writing** Consider the expression  $(1 + 5)^2 - (18 \div 3)$ . Can you perform the operations in different orders and still get the correct answer? Explain.

54. A student wrote the expressions shown and claimed they were equal for all values of  $x$  and  $y$ .

✓ a. Evaluate each expression for  $x = 1$  and  $y = 0$ .

✗ b. Evaluate each expression for  $x = 1$  and  $y = 2$ .

c. **Open-Ended** Choose another pair of values for  $x$  and  $y$ . Evaluate each expression for those values.

d. **Writing** Is the student's claim correct? Justify your answer.

55. Find the value of  $14 + 5 \cdot 3 - 3^2$ . Then change two operation signs so that the value of the expression is 8.

$$14 + 5 \cdot 3 - 3^2 = 8$$

$$14 + (5 \cdot 3 - 9) = 8$$

$$17 - 9 = 8$$

$$14 - 6 = 8$$

$$14 + 15 - 7$$

$$29 - 9 = 20$$

$$(1+0)^2 = 1^2 = 1 \quad 1^2 + 0^2 = 1 + 0 = 1$$

$$\frac{(x+y)^2}{x^2+y^2}$$

$$(1+2)^2 = 9 \quad 1^2 + 2^2 = 1 + 4 = 5$$


**Challenge**

Use grouping symbols to make each equation true.

$$56. (9 + 3) - (2 + 4) = 6$$

$$57. (16 - 4) \div 2 + 3 = 9$$

$$58. 4^2 - 5 \cdot 2 + 1 = 1$$

$$59. 3 \cdot (4 + 5) - 6 + 7 = 28$$

$$60. a. 16 - 5(2 + 1) = 1$$

60. a. **Geometry** A cone has a slant height  $\ell$  of 11 cm and a radius  $r$  of 3 cm. Use the expression  $\pi r(\ell + r)$  to find the surface area of the cone. Use 3.14 for  $\pi$ . Round to the nearest tenth of a square centimeter.

- b. **Reasoning** Does the surface area of the cone double if the radius doubles? If the slant height doubles? Explain.